



## SM358

# Frequently Asked Questions

Is the speed of a de Broglie wave equal to the speed of the corresponding particle?

*Surprisingly enough, the answer to this question turns out to be "no". This optional note explains why, and also discusses why this is not a problem for wave mechanics.*

The equations relating the energy  $E$  and momentum  $p$  of a free particle to the frequency  $f$  and wavelength  $\lambda$  of the corresponding de Broglie wave are as follows:

$$E = hf \quad \text{and} \quad p = \frac{h}{\lambda}.$$

We can use these equations to find the speed of a de Broglie wave using the standard formula  $v_{\text{wave}} = f\lambda$ . We get

$$v_{\text{wave}} = f\lambda = \frac{E}{h} \times \frac{h}{p} = \frac{E}{p}.$$

For any free particle,  $E = p^2/2m$ , so the speed of the de Broglie wave works out to be

$$v_{\text{wave}} = \frac{p}{2m}.$$

This is perhaps a surprise. Any non-relativistic particle obeys the formula  $p = mv$ , where  $v$  is the speed of the particle, so  $v = p/m$  and we cannot avoid the conclusion that

$$v_{\text{wave}} = \frac{1}{2}v.$$

*The speed of the de Broglie wave is half that of the corresponding particle!*

How can this be? Isn't it a problem for the Broglie wave to "lag behind" the particle it describes? Rather wonderfully, the problem evaporates.

First, note that a de Broglie wave refers to a highly idealized situation in which the particle has a definite momentum  $p$ . The de Broglie relationship then guarantees that the corresponding wave has a single definite wavelength  $\lambda = h/p$  – and this means that the wave extends to infinity. (Any wave-like disturbance occupying a finite region of space would be a superposition of waves of *different* wavelengths.) Since the de Broglie wave exists everywhere in this idealized case, there can be no question of the particle leaving the wave behind.

This may seem like a sneaky way out of the difficulty, so let's go a bit further. What if we consider a particle whose momentum is uncertain, though fairly well-defined? Later in the module you will see that such a particle is represented by a so-called *wave packet* – a superposition of waves of different wavelengths. Such a wave packet is negligible outside a finite region of space. Now there appears to be a danger that the particle will outpace the corresponding wave packet, so that the particle will eventually be in a region where the wave packet is negligible – a prediction that would be absurd enough to raise serious doubts about wave mechanics.

Fortunately, this outpacing does not happen. It turns out that the centre of activity of the wave-packet (where the oscillations have high amplitudes) moves with exactly the same speed as the particle. Individual waves of short wavelength move faster than average, and so move from the trailing edge of the wave packet towards the leading edge. By contrast, individual waves of long wavelength move slower than average, and move from the leading edge of the wave packet towards the trailing edge. As a result, the wave packet distorts in shape, stretching out as it travels, but its centre of activity always moves with the speed of the particle, which is the key point here. (Wave packets will be discussed in more detail in Chapter 6 of Book 1, including computer simulations of their motion.)

For completeness, here are some further mathematical details. In the physics of waves, the speed of an individual wave of a definite wavelength is always given by the formula  $v_{\text{wave}} = f\lambda$ , which can be rewritten as

$$v_{\text{wave}} = \frac{\omega}{k},$$

where  $\omega = 2\pi f$  is the angular frequency and  $k = 2\pi/\lambda$  is the wave number of the wave. This speed is more correctly called the *phase speed* of the wave.

By contrast, the centre of activity of a wave packet moves at a speed called the *group speed*, given by

$$v_{\text{group}} = \frac{\partial\omega}{\partial k}.$$

Now the standard equations  $E = hf$  and  $p = h/\lambda$  can be rewritten as

$$E = \hbar\omega \quad \text{and} \quad p = \hbar k$$

where  $\hbar = h/2\pi$ . We therefore have

$$v_{\text{group}} = \frac{\partial\omega}{\partial k} = \frac{\partial E}{\partial p}.$$

For a free particle we know that  $E = p^2/2m$ , so we obtain

$$v_{\text{group}} = \frac{1}{2m} \frac{\partial p^2}{\partial p} = \frac{p}{m},$$

which is equal to the speed of the particle, as we claimed.

You need not remember these details about phase and group speeds, which are not part of this module and are not assumed knowledge. One thing is worth remembering though: do not use the formula  $v = f\lambda$  to calculate the speed of a free particle. This formula gives the phase speed of a de Broglie wave, *which is not the same as the speed to the particle*. The speed of a particle is given by  $p/m$ , where  $p$  is the momentum of the particle which can be found from  $p = h/\lambda$  or from  $E = p^2/2m = hf$ .

Incidentally, the case of light waves is much less troublesome. The energy of a photon is *not* given by the formula  $E = p^2/2m$ . Clearly, this formula makes no sense for photons because they have  $m = 0$ . For photons, the formula  $E = pc$  applies instead, where  $c$  is the photon speed and  $p$  is the photon momentum.

If we now calculate the phase speed of the light wave, using the same methods as above, we get

$$v_{\text{wave}} = f\lambda = \frac{E}{p} = c.$$

So, for photons, the de Broglie wave (the light wave) moves at the same speed as the corresponding particle (the photon). Moreover, for photons, the group speed is

$$v_{\text{group}} = \frac{\partial\omega}{\partial k} = \frac{\partial E}{\partial p} = c,$$

which is the same as the phase speed, so wave packets of light also move at the speed of photons. There is no question of lagging behind in this case.